# QCD CORRECTIONS TO THE $H^+ \to t \overline{b}$ DECAY WITHIN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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#### **ABSTRACT**

I present the results of the QCD corrections to the  $H^+ \to t \overline{b}$  decay within the minimal supersymmetric standard model, if gluinos and scalar quarks are taken within the relevant loop diagram. I include the mixing of the scalar partners of the left- and right-handed top quark, which is proportional to the top quark mass. The standard corrections via gluons and quarks are about +8% for a charged Higgs mass of 300 GeV and about -11% for a Higgs mass of 800 GeV. I show that the standard corrections are diminished or enhanced by a non-negligible amount for certain values of the supersymmetric parameter space. I also obtain sign changes.

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#### I. INTRODUCTION

Recently there has been a lot of interest in the QCD loop corrections [1,2] as well as in the electroweek loop corrections [3,4] of the charged Higgs boson decay into a top quark and an anti-bottom quark.

The underlying models were the two Higgs doublet model [see ref.5 and references therein] and the minimal supersymmetric extension of the standard model (MSSM) [6,7]. In this Brief Report I take the last one as the underlying model to consider the QCD corrections to the  $H^+ \to t\bar{b}$  decay mode.

According to eq.19 in ref.1 the QCD corrections via gluons and quarks within the relevant loop diagram (which I call SMG from now on) do have both signs depending on the mass of the charged Higgs boson. With the recently released CDF value for the top quark mass [8] the ratio between the zeroth order loop correction to the first order  $\Gamma^1/\Gamma^0$  is about +7.5% for a charged Higgs mass of 300 GeV, -2.4% for a mass of 500 GeV and -10.6% for a mass of 800 GeV.

In this Brief Report I present the QCD loop corrections to the  $H^+ \to t\bar{b}$  decay if gluinos and scalar quarks are taken within the loop as shown in Fig.1. Throughout the calculations I neglect the mass of the bottom quark, but I do not neglect the mixing of the scalar partners of the left- and right-handed top quark, which is proportional to the top quark mass.

In the next section I present the results of the calculation of the Feynman diagram given in Fig.1 and in the last section I discuss the results and end with the conclusions.

## II. SUSY QCD CORRECTIONS TO $H^+ \to t \overline{b}$

In the following I proceed as I did in the calculation of the SUSY QCD corrections to the  $t \to H^+b$  decay presented in ref.9. Taking the tree level coupling of the charged Higgs to a top and bottom quark given in eq.(1) in ref.9 the zeroth order to the decay rate  $\Gamma(H^+ \to t\bar{b})$  is given by:

$$\Gamma^{0}(H^{+} \to t\overline{b}) = \frac{G_{F}}{\sqrt{2}} |V_{tb}^{KM}|^{2} \cot^{2} \beta \frac{1}{4\pi} m_{H^{+}} m_{\text{top}}^{2} \left(1 - \frac{m_{\text{top}}^{2}}{m_{H^{+}}^{2}}\right)^{2}$$
(1)

 $\cot \beta = v_1/v_2$  is the ratio of the vacuum expectation values (vev) of the two Higgs doublets and  $V_{tb}^{KM} \approx 1$  the value of the Kobayashi Maskawa matrix. The calculation of the Feynman diagram in Fig.1 is similar as presented in ref.9. As a result I get for the first order in  $\alpha_s$ :

$$\Gamma'^{1}(H^{+} \to t\bar{b}) = \Gamma^{0}(H^{+} \to t\bar{b}) \left[ 1 - \frac{2\alpha_{s}}{3\pi} (S + A) \right]$$

$$S = S_{t} + \frac{m_{\tilde{g}}}{m_{\text{top}}} S_{\tilde{g}}$$

$$A = A_{t} + \frac{m_{\tilde{g}}}{m_{\text{top}}} A_{\tilde{g}}$$
(2)

$$S_{t} = K_{11} \left[ c_{\Theta}^{2} C_{1}^{\tilde{b}_{1}\tilde{t}_{1}} + s_{\Theta}^{2} C_{1}^{\tilde{b}_{1}\tilde{t}_{2}} \right] + K_{21} \left[ s_{\Theta} c_{\Theta} \left( C_{1}^{\tilde{b}_{1}\tilde{t}_{1}} - C_{1}^{\tilde{b}_{1}\tilde{t}_{2}} \right) \right]$$

$$A_{t} = S_{t}$$

$$S_{\tilde{g}} = K_{11} \left[ c_{\Theta} s_{\Theta} \left( C_{0}^{\tilde{b}_{1}\tilde{t}_{2}} - C_{0}^{\tilde{b}_{1}\tilde{t}_{1}} \right) \right] - K_{21} \left[ c_{\Theta}^{2} C_{0}^{\tilde{b}_{1}\tilde{t}_{2}} + s_{\Theta}^{2} C_{0}^{\tilde{b}_{1}\tilde{t}_{1}} \right]$$

$$A_{\tilde{g}} = S_{\tilde{g}}$$

$$K_{11} = 1 - \frac{m_{W}^{2}}{m_{\text{top}}^{2}} \tan \beta \sin 2\beta$$

$$K_{21} = \frac{1}{m_{\text{top}}} \left( A_{\text{top}} + \mu \tan \beta \right)$$

$$\begin{split} C_0^{\tilde{b}_j \tilde{t}_i} &= -\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m_{\text{top}}^2}{f_{\tilde{g}}^{\tilde{b}_j \tilde{t}_i}} \\ C_1^{\tilde{b}_j \tilde{t}_i} &= -\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m_{\text{top}}^2 \alpha_1}{f_{\tilde{g}}^{\tilde{b}_j \tilde{t}_i}} \\ f_{\tilde{g}}^{\tilde{b}_j \tilde{t}_i} &= m_{\tilde{g}}^2 - (m_{\tilde{g}}^2 - m_{\tilde{t}_i}^2)\alpha_1 - (m_{\tilde{g}}^2 - m_{\tilde{b}_j}^2)\alpha_2 - m_{\text{top}}^2 \alpha_1 (1 - \alpha_1 - \alpha_2) - m_{H^+}^2 \alpha_1 \alpha_2 \end{split}$$

where  $c_{\Theta} = \cos \Theta$  and  $s_{\Theta} = \sin \Theta$  is the mixing angle of the scalar top quarks with the following mass matrix:

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_{\text{top}}^2 + 0.35D_Z^2 & -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) \\ -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) & m_{\tilde{t}_R}^2 + m_{\text{top}}^2 + 0.16D_Z^2 \end{pmatrix}$$
(3)

where  $D_Z^2 = m_Z^2 \cos 2\beta$ .  $m_{\tilde{t}_{L,R}}^2$  are soft breaking masses,  $A_{\text{top}}$  is the trilinear scalar interaction parameter and  $\mu$  is the supersymmetric mass mixing term of the Higgs bosons.

The mass eigenstates  $\tilde{t}_1$  and  $\tilde{t}_2$  are related to the current eigenstates  $\tilde{t}_L$  and  $\tilde{t}_R$  by  $\tilde{t}_1 = \cos\Theta\tilde{t}_L + \sin\Theta\tilde{t}_R$  and  $\tilde{t}_2 = \cos\Theta\tilde{t}_L - \sin\Theta\tilde{t}_R$ . The mass eigenstates are functions of the scalar breaking mass term  $m_S$  as well as of  $A_{\rm top}$  and  $\mu$ . In a global SUSY model we have  $A_{\rm top} = m_S$  and for neglecting bottom quark masses we have  $m_{\tilde{b}_1}^2 = m_S^2 - 0.42 m_Z^2 \cos 2\beta$  and  $m_{\tilde{b}_2}^2 = m_S^2 - 0.08 m_Z^2 \cos 2\beta$ . With neglecting bottom quark mass the scalar partners of the left and right handed bottom quarks do not mix and therefore  $m_{\tilde{b}_L} = m_{\tilde{b}_1}$ .

The S and A in eq.(2) indicate that the contribution comes from the scalar and axial scalar coupling of the matrix element. In the case of no mixing of the scalar top quarks the gluino terms  $S_{\tilde{g}}$  and  $A_{\tilde{g}}$  do not contribute  $(K_{21} = 0 = s_{\Theta})$ . The Feynman integration can be done numerically.

In eq.(19) in ref.1 the authors present the results of the standard QCD one loop corrections, which I have to include in my calculation. As a final result I obtain:

$$\Gamma^{1}(H^{+} \to t\overline{b}) = \Gamma^{0}(H^{+} \to t\overline{b}) \left[ 1 + \frac{2\alpha_{s}}{3\pi} (G_{SM} - (S+A)) \right]$$

$$\tag{4}$$

$$G_{SM} = 3\ln(1 - \beta_t) - 2\frac{(1 - \beta_t^2)}{\beta_t} \ln(1 - \beta_t) + 2\ln(1 - \beta_t) \ln(\beta_t)$$
$$- (3 + 2\beta_t) \ln(\beta_t) + \frac{1}{2} \text{Li}_2((1 - \beta_t)^2) + 3\text{Li}_2(1 - \beta_t)$$
$$- 4\text{Li}_2((1 - \beta_t)^{1/2}) - \frac{2}{\beta_t} + \frac{13}{2}$$
$$\beta_t = 1 - \frac{m_{\text{top}}^2}{m_{H^+}^2}$$

In the next section I discuss the results.

#### III. DISCUSSIONS

To compare the standard QCD correction given in ref.1 with the gluino and scalar contribution via eq.(2) I present in Fig.2–5 the results of  $\Gamma^1/\Gamma^0$  for different cases of the charged Higgs mass,  $\tan \beta$  and the gluino mass as a function of the scalar mass  $m_S$ . Throughout the calculation I use 174 GeV for the top quark mass and  $\mu = 500$  GeV with  $A_{\rm top} = m_S$ . In Fig.2 and Fig.3 I consider a charged Higgs mass of 300 GeV and in Fig.4 and Fig.5 I take a charged Higgs mass of 800 GeV. I use  $\tan \beta = 1$  in Fig.2 and Fig.4 and  $\tan \beta = 2$  in Fig.3 and Fig.5. The results are presented for three different values of the gluino mass 3 GeV (solid line), 100 GeV (dotted line) and 500 GeV (dash-dotted line). The solid straight line is the SMG contribution.

One can see, that the gluino and scalar quark contribution can change the SMG contribution drastically and even lead to sign changes for certain values of the gluino and scalar masses. In Fig.2 and Fig.4 the variation of the  $\mu$  hardly affects the results, whereas in Fig.3 and Fig.5 its influence is much bigger due to a  $\mu \tan \beta$  dependence in the couplings. For higher values of  $\tan \beta$  the results are pushed farther away from the SMG result, although in this case as already was mentioned in ref.9, the bottom quark mass might become important. For  $m_{H^+}=800$  GeV the ratio  $\Gamma^1/\Gamma^0$  is decreasing again if  $m_S$  is larger than 500 GeV.

For  $\tan \beta = 1$  the heavier scalar top quark masses vary from 358 GeV to 631 GeV and the lighter scalar top quark mass is about 250 GeV for  $m_S$  smaller than 100 GeV, decreases constantly to 70 GeV for  $m_S = 350$  GeV and increases again to 260 GeV in the range considered here. The scalar bottom masses are equal to  $m_S$ . For  $\tan \beta = 2$  the heavier scalar top quark masses vary from 289 GeV to 594 GeV, the lighter scalar top quark mass is about 145 GeV for  $m_S$  smaller than 100 GeV, decreases to 67 GeV for  $m_S = 250$  GeV and increases to 331 GeV for  $m_S = 450$  GeV. The heavier scalar bottom quark mass vary from 68 GeV to 452 GeV and the lighter one from 54 GeV to 450 GeV in the range of the scalar mass  $m_S$  considered here. In both cases  $\cos \Theta \approx 1/\sqrt{2}$ .

#### III. CONCLUSIONS

In this Brief Report I have compared the first order in  $\alpha_s$  contribution of the

gluon and quarks to the deacay rate  $\Gamma(H^+ \to t\bar{b})$  with the contribution of gluino and scalar quarks to this decay rate. I have shown that the contribution of scalar quarks and the gluino are not negligible and changes the SMG contribution drastically and even leads to different signs for certain values of the SUSY masses.

Finally I want to mention, that the electroweak corrections with a top quark within the loop can have a relative sign compared to the QCD corrections depending on the charged Higgs mass and therefore also might be important; this was shown in ref.3. Here in this Brief Report I have shown that a gluino mass of 500 GeV contributes the most to the  $H^+ \to t \bar b$  decay rate and gets even more important if  $\tan \beta$  increases whereas according to ref.3 the electroweak top quark contribution decreases.

#### IV. ACKNOWLEDGMENTS

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### FIGURE CAPTIONS

- Fig.1 The diagram with scalar quarks and gluino within the loop, which contribute to the charged Higgs boson decay into a top and antibottom quark.
- Fig.2 The ratio of  $\Gamma^1/\Gamma^0$  as a function of the scalar mass  $m_S$  for 3 different values of the gluino mass: 3 GeV (solid line), 100 GeV (dotted line) and 500 GeV (dash-dotted line) with  $\mu = 500$  GeV and  $A_{\text{top}} = m_S$ .  $v_1 = v_2$  and  $m_{H^+} = 300$  GeV. The solid straight line is the SMG contribution as given in eq.(19) in [1].
- Fig.3 The same as Fig.2 with  $v_2 = 2 \cdot v_1$ .
- Fig.4 The same as Fig.2 with  $m_{H^+} = 800$  GeV.
- Fig.5 The same as Fig.4 with  $v_2 = 2 \cdot v_1$ .

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